KINEMATIC ANALYSIS OF PRINTING MATERIALS CUTTING USING CIRCULAR CUTTERS

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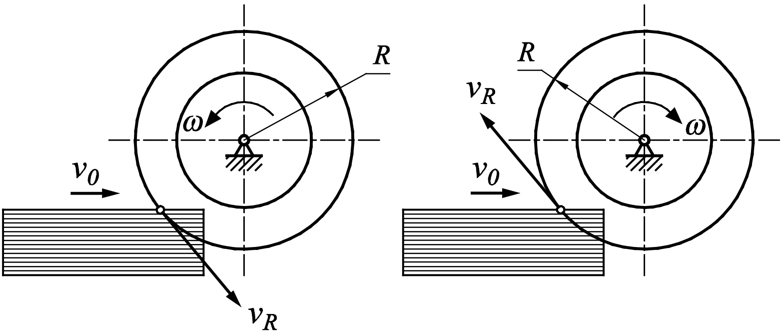
Abstract
The cutting process using circular cutters is most widespread among different cardboard and paper cutout methods. The circular cutters are commonly used to cut materials in rolls and sheets such as: foils, papers with different density or thick cardboards. The process of cutting depends on many factors such as cut material, cutter material, speed of process and geometric parameters of cutting action. One of the most important factors having impact on the cutting process is the cutter sharpening angle, or edge angle. Minimizing of cutter sharpening angle decreases the cutting force, but the minimal possible edge angle is strongly restricted by the mechanical properties of cutter material. During the cutting, a kinematic transformation of the edge angle occurs. This transformation depends on kinematic and geometrical parameters of the cutting process: cutter angular speed and rotation direction, feed rate, cutter diameter and material thickness. The effective cutting angle (the kinematic one) is reduced comparable to the sharpened edge angle. This causes a significant decreasing of cutting forces. The main goal of this investigation is to analyze the transformation of the circular cutter blade-sharpening angle into the effective cutting angle during a cutting action.

The cutting process using circular cutters is the most widespread among different cardboard and paper cut out methods. The circular cutters are commonly used to cut materials in rolls and sheets such as foils, papers with different density or thick cardboards. The first reason why this study was taken was to find out the best possible geometric parameters of the circular cutter. This helps to decrease cutting forces and to find the lowest possible kinematic edge angle of the cutter α_T. That is so important because of the fact that rising the static edge angle of the cutter goes up and improves the strength and durability of the cutter.

During the process of cutting using rotating cutter, a kinematic transformation of the edge angle occurs; the mentioned transformation depends on kinematic and geometrical parameters of the cutting process as: cutter angular speed and rotation direction, feed rate, cutter diameter and material thickness. The effective cutting angle value (the kinematic one) becomes smaller than actual sharpened edge angle. It also causes a significant decreasing of cutting forces.

The analysis has been carried out for two rotation directions of the cutter: synchronous – takes place when the cutter rotation direction is synchronous to the movement direction of the workpiece (Fig.1a) and counter-synchronous (anti) – takes place when the cutter rotation direction is anti-synchronous to the treated material (Fig.1b).

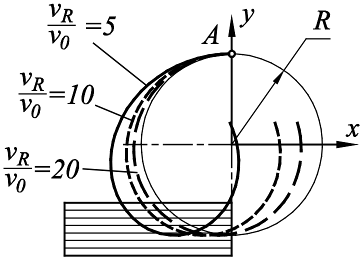


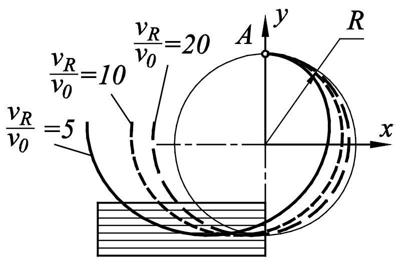
a) Synchronous cutting *b) Anti-synchronous cutting*

Fig.1. The schemes of cutting processes with circular cutters

R – radius of circular cutter
v_0 – feed rate of a workpiece (paper pile)
ω – circular cutter rotation Speed
$v_R = \omega R$ – linear velocity of a point on an knife edge

In order to study the movement trajectory of a cutter edge point with relation to a workpiece, we used a method of reversed motion. Supposing the rotating knife is moving towards the fixed workpiece, the velocity of the knife's center equals to the feed rate v_0. In this case, the parametric equations of trajectory of a random edge point A are: $x_A = -v_0 t - R \cdot \sin(\omega t)$ $y_A = R \cdot \cos(\omega t)$ – for the synchronous cutting and $x_A = -v_0 t - R \cdot \sin(\omega t)$ $y_A = R \cdot \cos(\omega t)$– for the counter-synchronous cutting, where t – time. Fig.2 shows the movement trajectory of a point from the edge of cutter. Fig.2a shows synchronous movement direction, fig.2b shows the opposite movement direction for different speed ratio (v_R/v_0). The trajectory of the edge point movement for both situations is an elongated cycloid. The form of the trajectory depends on the linear velocity v_R to feed speed v_0 ratio. Rising the ratio makes movement trajectory shape more similar to a circle.





a) Synchronous cutting *b) Anti-synchronous cutting*

Fig. 2. Movement trajectories of the point at the edge of cutter

During the cutting process according to calculated trajectory, the further kinematic transformation of the edge angle occurs. The effective cutting angle (kinematic angle) is reduced comparable to the sharpened edge angle.

The kinematic of cutting using circular knife differs from the kinematic process with regular flat knife cutter. The actual cutting angle changes not significant using regular flat cutter with one knife cutting machine. The transformed (or effective) edge angle depends on vertical and horizontal velocity of a cutter and on sharpening angle only. During the cutting process with circular cutters, the sharpening angle depends on kinematic parameters of treatment and on the shape of cutting edge. In that situation, different layers of a workpiece are being cut by different sharpening angle of the circular cutter.

When cutting with regular flat knife cutter (Fig.3a), the actual cutting angle is calculated with the following formula [4]:

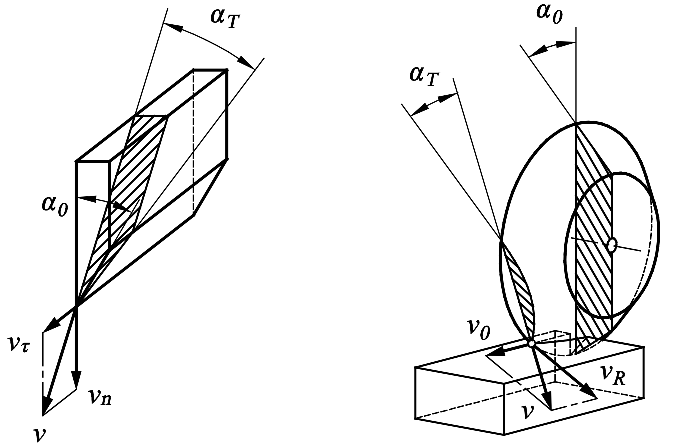
$$\alpha_T = \arctan\left(\frac{\tan(\alpha_0)\cdot v_n}{v}\right) \quad (1)$$

where:

α_0 – static sharpening angle,

v – full velocity of cutting process,

v_n – normal velocity component.



a) Flat cutter *b) Circular cutter*

Fig. 3. The scheme of a knife sharpening edge transformation.

α_0 – knife-sharpening angle (static one)
α_T – actual knife sharpening angle (kinematic transformation)
v – full velocity of cutting process
v_n, v_τ – normal and tangential components of cutting velocity
v_0 – the feed rate
v_R – linear velocity of the cutting edge

Suppose, kinematic transformation of the cutting angle of the circular cutter is similar to the transformation of a flat knife [2,3]. To obtain the actual cutting angle α_T for synchronous cutting (Fig. 1a), we can calculate the velocity components from Fig.4b and apply appropriate substitutions to the expression (1). The resulting expression is formula (2). The resulting expression for anti-synchronous cutting (Fig.1b) is the formula (3):

$$\alpha_T = \arctan\left[\tan(\alpha_0)\frac{v_0\cos(\varphi)}{\sqrt{[v_R\sin(\varphi)-v_0]^2+[v_R\cos(\varphi)]^2}}\right] \quad (2)$$

Synchronous cutting

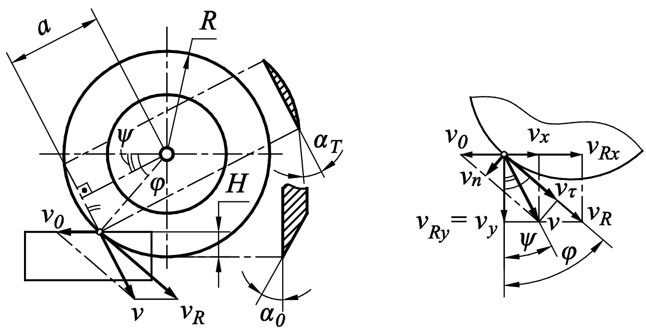
$$\alpha_T = \arctan\left[\tan(\alpha_0)\frac{v_0\cos(\varphi)}{\sqrt{[v_R\sin(\varphi)+v_0]^2+[v_R\cos(\varphi)]^2}}\right] \quad (3)$$

Anti-synchronous cutting

where:

$$\varphi = \arcsin(R - H)/R \quad \text{(Fig. 4a).}$$

In the majority of paper cutting processes using circular cutters, the process takes place at the edges of circular cutters. The sharpening angle of circular knife blade α_T is defined inside a slant cross-section (Fig.3b). Taking into account the conical surface of the cutting edge, the cross-section represents a flat shape with a sharpening angle α_T, bounded with a hyperbola and a straight line (Fig.4a). This shape differs from a triangle shape of a flat knife cross-section (Fig.3a).



a) Transformation of the sharpening angle at the edge of the cutter. *b) The velocity distribution at the cutter edge.*

Fig. 4. Calculating schemes for circular cutter edge angle transformation for synchronous cutting.

α_0 – knife sharpening angle (static one),
α_T – actual knife sharpening angle (kinematic one),
v – full speed of cutting process,
v_n, v_τ – normal and tangential components of cutting velocity,
v_0 – the movement speed of workpiece,
v_R – linear velocity of the rotating cutting edge,
v_{Rx} i v_{Ry} – linear velocity components,
H – depth of cutting.

In order to refine the results, we calculated the transformed angle using the hyperbola equation and its derivative [1]. The appropriate formulas for the synchronous cutting are:

$$\alpha_T = \arctan\left(\tan(\alpha_0)\frac{\sqrt{R^2 - a^2}}{R}\right) =$$

$$= \arctan\left\{\tan(\alpha_0)\frac{\sqrt{R^2 - R^2\cos^2\left[\varphi - \arctan\left(\frac{v_R\sin(\varphi) - v_0}{v_R\cos(\varphi)}\right)\right]}}{R}\right\} \quad (4)$$

where:

$$a = R\cos(\varphi - \psi)$$

$$\psi = \arctan\left(\frac{v_{Rx} - v_0}{v_{Ry}}\right) = \arctan\left(\frac{v_R\sin(\varphi) - v_0}{v_R\cos(\varphi)}\right)$$

The further transformation of the expression (4) with respect to geometrical and kinematical relations leads to the same formula as (2). The similar result is obtained for anti-synchronous cutting. This allows using of the same expressions to calculate actual edge angle for circular cutters and for flat knives. A feature of the cutting with circular cutters is that the velocity distribution at the edge depends on the cutting depth *H* (Fig.4a) and changes along the cutter circular blade. The actual sharpening angle changes too. Different layers of paper are being cut by another actual sharpening angle of the blade. The largest value of the actual sharpening angles occurs at the extreme point of contact of the blade with a workpiece. When cutting using synchronous method, the extreme point takes place where knife enters into workpiece. When using counter- synchronous method, the extreme point takes place where the knife exits from a workpiece. Using the formulas (2, 3), the kinematic sharpening cutter edge angle α_T for different cutting parameters was calculated. The result of calculation is that the value of the kinematic edge angle for anti-synchronous cutter rotation is slightly lower than the value for the synchronous rotation direction by the same geometric parameters of circular cutters. The highest impact for the above situation the following parameters of cutting process have: diameter of cutter, rotation speed, the speed of workpiece movement (feed rate) and the depth of cutting (Fig. 5, 6). The curves are built for the following cutting parameters: cutter radius *R*=100 mm, static sharpening angle α_0=15°, cutting depth *H*=2.5; 5; 10; 20; 30; 50 mm (bottom to top curves).

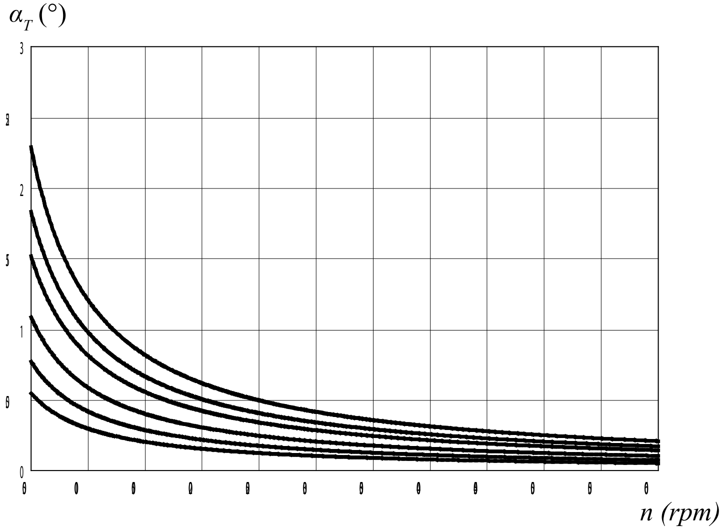


Fig. 5. Kinematic reduction of sharpening edge angle α_T by different rotation speeds n. Counter-synchronous cutting. v_0=1000mm/sec.

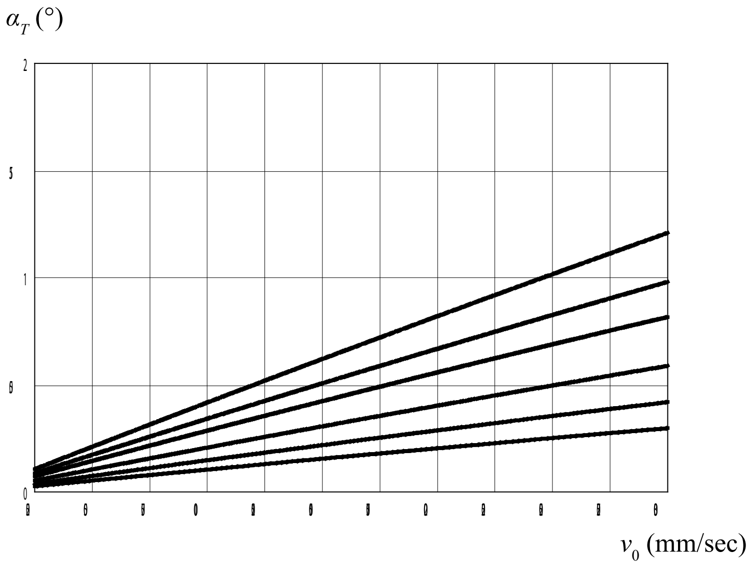


Fig. 6. Kinematic reduction of sharpening edge angle α_T by different feed rates v_0. Counter-synchronous cutting. n=3000rpm.

Analysing the results of calculations it's easy to observe that the direction of cutter rotation during cutting process dosen't have meaningful impact on the kinematic edge angle transformation. On the other hand, increasing of a circular cutter radius *R* decreases the kinematic edge angle α_T. The impact is significat with rising the radius value up to 100mm, beyond that size the change stops to be meaning. Moreover, rising up the rotation speed of the cutter lowers the kinematic edge angle α_T, but this takes action only within the range *n* = 1000…3000 rpm. Further rising the rpm parameter does not have any significant influence. The analysis of the workpiece speed movement proved to be in a quasi-linear subjection with the change of the kinematic angle edge $\alpha_{T.}$.

Reference list

1. Janicki P.; Petriaszwili G., Uwzględnienie kształtu krawędzi tnącej noża krążkowego przy obliczeniach kinematyki procesu krojenia papieru, *Opakowanie*, no. 12, 2015, p. 66-68, ISSN 0030-3348
2. Janicki P.; Petriaszwili G., Transformacja kinematycznego kąta zaostrzenia ostrza noża w procesach rozkroju tektury i papieru nożami krazkowymi. *Opakowanie,* no. 9, 2015, p. 79-81, ISSN 0030-3348.
3. Petriaszwili G., Analiza kinematyki procesów krojenia papieru i tektury wibracyjnymi nożami krązkowymi. *Opakowanie*, no. 12, 2014, p. 55-57, ISSN 0030-3348.
4. Mordowin B. M. *Buchbindereimaschinen - Berechnung und Konstruktion polygraphischer Maschinen*, I Band. Verlag Technik Berlin, 1962.